Optimal Control Problems for Elliptic Hemivariational Inequalities

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We consider a bounded domain Ω in $\mathbb{R}^d$ whose regular boundary Γ consist of the union of three disjoint portions Γ_i, $i = 1, 2, 3$ with $meas(\Gamma_i) > 0$. We formulate the following nonlinear elliptic problem with mixed boundary conditions [3]:

$$-\Delta u = g \text{ in } \Omega, \quad u\big|_{\Gamma_1} = 0, \quad -\frac{\partial u}{\partial n}\big|_{\Gamma_2} = q, \quad -\frac{\partial u}{\partial n}\big|_{\Gamma_3} \in \alpha\, \partial j(u), \tag{1}$$

where α is a positive constant, $g \in L^2(\Omega)$, $q \in L^2(\Gamma_2)$ and the function $j\colon \Gamma_3 \times \mathbb{R} \to \mathbb{R}$, called a superpotential (nonconvex potential), is such that $j(x, \cdot)$ is localy Lipschitz for a.e. $x \in \Gamma_3$ and not necessary differentiable. Such multivalued condition on Γ_3 is denoted for a nonmonotone relation expressed by the generalized gradient of Clarke [2]. The weak formulation of (1) is given by the elliptic hemivariational inequality [3, 5]:

$$\text{find } u \in V_0 \text{ such that } a(u, v) + \alpha \int_{\Gamma_3} j^0(u; v)\, d\Gamma \geq L(v), \quad \forall v \in V_0, \tag{2}$$

where j^0 represent the generalized (Clarke) directional derivative, $a(u, v) = \int_\Omega \nabla u \,\nabla v\, dx$, $L(v) = \int_\Omega gv\, dx - \int_{\Gamma_2} qv\, d\gamma$ and $V_0 = \{v \in H^1(\Omega) : v = 0 \text{ on } \Gamma_1\}$.

We formulate for each $\alpha > 0$, different optimal control problems (C_α), on the internal energy g and the heat flux q, for quadratic cost functional and we prove existence results for the optimal solutions (see [1, 4]). We also consider a problem as (1), with a Dirichlet condition on Γ_3 and we formulate similar optimal control problems (C), on control variables g and q. We obtain, convergence results for optimal controls and system states (C_α) to the corresponding optimal control and system state (C), when the parameter α goes to infinity.

Bibliography

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